PMM U.S.S.R.,Vol.51,No.3,pp.386-392,1987
Printed in Great Britain
0021-8928/87 $10.00+0.00

ON THE SHAPE OF MINIMUM-RESISTANCE SOLIDS OF REVOLUTION MOVING IN PLASTICALLY COMPRESSIBLE AND ELASTIC-PLASTIC MEDIA*

A.I. BUNIMOVICH and G.E. YAKUNINA

A variational problem on the shape of minimum-drag thin solids of revolution moving at a constant velocity in plastically compressible and elastic-plastic media simulating soil and metals, respectively, is formulated and solved under the assumption of the validity of the plane section hypothesis. Optimal solid of revolution shapes are found when there is friction in the medium. Optimal body shapes and their resistance are computed on a computer for real soils under different isoperimetric conditions. It is shown that a cone is not the optimal solid in a number of cases. One special case of the problem being investigated was examined in /1/ in an analogous formulation.

1. Formulation of the problem. A body moving in a medium overcomes the resistance this medium. To ensure maximum depth of penetration it is necessary, other conditions being equal, to select that body shape that will ensure body motion for minimum possible resistance of the medium. We will consider plastically compressible media which, with an appropriate selection of the governing parameters, will simulate real media, soils in particular. We will assume that the body is thin and moves at constant velocity.

Because the velocity of thin body penetration substantially exceeds the velocity of particle motion of the medium under consideration and the interaction of the body with the medium is inelastic, it can be assumed that the moving body seems to "displace" aside the particles of the medium, i.e, the "plane sections" hypothesis is valid /2/.

W will confine ourselves to considering solids of revolution. Let the body axis be denoted by the x axis in a cylindrical system of coordinates. We give the generator of the solid of revolution in the form

$$y = f(x) \tag{1.1}$$

where $f(x)$ is a twice-differentiable function satisfying the convex body conditions

$$f'(x) = df/dx \geqslant 0, \; f''(x) = d^2f/dx^2 < 0 \tag{1.2}$$

Under the assumptions made, the resistance force D acting on a thin solid of revolution of length $x_k = L$ penetrating into the plastically compressible medium at a constant velocity u directed along the body axis determined by the functional /3/

$$D = 2\pi \int_0^{x_k} p_0 (f' + \mu_0) f \, dx \tag{1.3}$$

$$p_0 = Af'^2 + Bff'' + G \tag{1.4}$$

Here p_0 is the pressure of the medium on the surface of the penetrating body, μ_0 is the coefficient of dry friction, and $A \geqslant 0, \; B \geqslant 0, \; G \geqslant 0$ are coefficients governing the properties of the medium.

A shock wave moves ahead of the body as the body moves in the medium. Consequently, the coefficients A, B, G should depend on the initial density ρ_0 of the medium, the density ρ of the medium behind the shock preceding the body, the magnitude m of the adhesion, the angle θ of internal friction, and the counterpressure p_a of the medium.

Introducing the notation

$$\begin{gathered} b_1 = \rho_0/\rho_1 = \text{const}, \; a = (1 - b_1)^{-1} \\ \tau_0 = 2m \cos\theta, \; \varphi = \sin\theta, \; \gamma = \varphi/(1 + \varphi) \\ \rho_0 u^2 = 2\chi, \; \rho_1 u^2 = 2\chi_1 \end{gathered} \tag{1.5}$$

we will write the expression for the coefficients A, B, G of the medium in the form utilized most often in practical computations /3/

*Prikl.Matem.Mekhan.,51,3,496-503,1987

$$A=\begin{cases}\frac{\chi_1}{1-\gamma}\left[a^{\gamma-1}-1+2b_1(1-\gamma)a^\gamma+\frac{1-\gamma}{\gamma}(a^\gamma-1),\quad \gamma\neq 0\right.\\ \chi_1(\ln a+b_1),\quad \gamma=0\end{cases} \tag{1.6}$$

$$B=\begin{cases}2\chi_1\gamma^{-1}(a^\gamma-1), & \gamma\neq 0\\ \chi_1\ln a, & \gamma=0\end{cases},\quad G=\begin{cases}(p_a+\tau_0\varphi^{-1}(a^\gamma-1), & \gamma\neq 0\\ \tau_0\ln a, & \gamma=0\end{cases}$$

Let us determine the class of allowable functions $f(x)$. Since thin bodies are considered, the condition $f'^2 \ll 1$ should be satisfied, i.e.,

$$f' \leqslant k_0,\quad k_0^2 = o(1) = \text{const} \tag{1.7}$$

For the model used for the medium, by considering separation-free flow around the bodies we obtain a constraint on the class of allowable functions from (1.4)

$$p_0 = Af'^2 + Bff'' + G \geqslant 0 \tag{1.8}$$

If $f'(x) \neq 0$, then the side surface S of the penetrating body and its volume V can be written in the form

$$\frac{S}{2\pi}=\int_0^{x_k} f\,dx=\int_0^{y_k} f x'\,df \tag{1.9}$$

$$\frac{V}{\pi}=\int_0^{x_k} f^2\,dx=\int_0^{y_k} f^2x'\,df;\quad x'=\frac{dx}{df}=\frac{1}{f'}$$

where $y_x = d/2 = R_0$ is the radius of the body base (Fig.1).

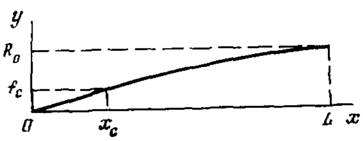

Fig.1

We assume that

$$f(0) = 0 \tag{1.10}$$

Therefore, the optimal body contour is sought in the class of allowable functions satisfying conditions (1.2), (1.7), (1.8)-(1.10). It is convenient to express conditions (1.2), (1.8) and (1.10) in the form

$$f' - l^2 = 0,\quad g' + w^2 = 0 \tag{1.11}$$

$$Ag^2 + Bfg' + G - \beta^2 = 0,\quad f' - k_0 + \alpha^2 = 0$$

where l, w, α, β are real variables and the function $g(x)$ is determined by the differential condition

$$f' - g = 0 \tag{1.12}$$

Therefore, the problem of determining the shape of a thin minimum-resistance body penetrating a plastically compressible medium at a constant velocity reduces to a variational problem to determine the functions $f(x), g(x), \alpha(x), w(x), l(x), \beta(x)$ minimizing the functional (1.3) and satisfying the isoperimetric conditions (1.9), the differential conditions (1.11) and (1.12), and given conditions at the ends.

2. Ceneral solution of the variational problem. The problem formulated in Sect.1 is equivalent to seeking the extremum of the functional

$$I=\int_0^{x_k} F(f, f', g, g', w, \alpha, l, \beta, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7)\,dx \tag{2.1}$$

under the conditions (1.11), (1.12), (1.9) and conditions at the ends. Here

$$F = (Ag^2 + Bfg' + G)(g + \mu_0)f + \lambda_3(x)(f' - g) + \lambda_4(x)(g' + w^2) + \lambda_5(x)(f' - k_0 + \alpha^2) + \lambda_6(x)(Ag^2 + Bfg' + G - \beta^2) + \lambda_7(x)(f' - l^2) + \lambda_1 f + \lambda_2 f^2 \tag{2.2}$$

($\lambda_1, \lambda_2, \lambda_3(x), \ldots, \lambda_7(x)$ are Lagrange multipliers).

The transversality conditions for the problem under consideration are written in the form

$$[F - f'F_{f'} - g'F_{g'}]_k\,\delta x_k + [F_{f'}]_k\,\delta y_k + [Fg']_k\,\delta g_k - [Fg']_0\,\delta g_0 = 0 \tag{2.3}$$

$$F_{f'} = \partial F/\partial f',\quad F_{g'} = \partial F/\partial g'$$

where the subscripts o and k correspond to values of the functions and variations at the initial and terminal points of the curve, respectively. Condition (2.3) should be satisfied for any allowable variations $\delta x_k, \delta y_k, \delta g_k$. Consequently, if additional constraints are not imposed on them, it follows that

$$[F - f'F_{f'} - g'F_{g'}]_k = 0, \quad [F_{f'}]_k = 0, \quad [F_{g'}]_k = 0, \quad [F_{g'}]_0 = 0 \tag{2.4}$$

By the condition of the problem no constraints are imposed on the variation δg at the initial and terminal points; therefore, the last two relationships in (2.4) should always be satisfied. They have the form

$$Bf_k^2 (f_k' + \mu_0) + \lambda_{4_k} + \lambda_{6_k} + Bf_k = 0, \ \lambda_{4_0} = 0 \tag{2.5}$$

Necessary conditions for a minimum. By applying the rule for solving problems with constraints of the equality and inequality type /4/, we obtain that the conditions

$$\lambda_4(x) \geqslant 0, \ \lambda_5(x) \geqslant 0, \ \lambda_6(x) \leqslant 0, \ \lambda_7(x) \leqslant 0 \tag{2.6}$$

which can also be obtained directly from the necessary condition for a minimum, the Legendre condition, should be satisfied along the extremal.

The Euler equation for the problem under consideration has the form

$$dF_{g'}/dx - F_g = 0, \ dF_{f'}/dx - F_f = 0 \tag{2.7}$$
$$F_\alpha = 0, \ F_\beta = 0, \ F_l = 0, \ F_w = 0$$

The last four relationships yield

$$\lambda_5\alpha = 0, \ \lambda_6\beta = 0, \ \lambda_4 w = 0, \ \lambda_7 l = 0 \tag{2.8}$$

We note that if $\lambda_4 \not\equiv 0$ along an arc of the extremum, then $\lambda_6 \equiv 0$ and, conversely if $\lambda_6 \not\equiv 0$, then $\lambda_4 \equiv 0$.

It follows from (2.5) and (2.6) that

$$\lambda_{6_k} = 0, \quad \text{if} \quad f_k = 0, \ \mu_0 > 0 \tag{2.9}$$

(since the extremal is the line $f \equiv 0$ in this case)

$$\lambda_{6_k} \neq 0, \quad \text{if} \quad f_k \neq 0, \ \mu_0 > 0 \tag{2.10}$$

(since otherwise $\lambda_{4_k} = -Bf_k^2, \ (f_k' + \mu_0) < 0$).

When conditions (2.10) is satisfied we obtain from (2.8)

$$\beta_k = 0 \tag{2.11}$$

On the basis of (1.11) in particular, condition (2.11) means that the extremal does not contain the segment $f' = 0$. But we then obtain from (1.11) and (2.8)

$$\lambda_7(x) \equiv 0 \tag{2.12}$$

Taking account of (2.12), we rewrite the first two Euler Eqs.(2.7):

$$\lambda_3 + \lambda_4' + \lambda_6' Bf - \lambda_6 g(2A - B) = (3A - 2B)g^2 f + \tag{2.13}$$
$$2\mu_0 (A - B)gf + Gf$$
$$d(\lambda_3 + \lambda_5)/dx = (Ag^2 + 2Bfg' + G)(g + \mu_0) + \lambda_6 Bg' +$$
$$\lambda_1 + 2\lambda_2 f$$

The function F does not contain the variable x explicitly, consequently, a first integral exists for the Euler Eq.(2.7)

$$F - f'F_{f'} - g'F_{g'} = c_1 = \text{const} \tag{2.14}$$

or in expanded form

$$(Ag^2 + G)(g + \mu_0)f = -\lambda_6 (Ag^2 + G) + \lambda_3 g + \lambda_5 k_0 + c_1 - \lambda_1 f - \lambda_2 f^2 \tag{2.15}$$

On the basis of (2.15) and the first of relationships (2.13), taking (1.12) into account, we obtain the following fundamental equation of the problem

$$[2(A - B)f'^3 + \mu_0 (A - 2B)f'^2 - \mu_0 G]f = -c_1 + \tag{2.16}$$
$$\lambda_1 f + \lambda_2 f^2 + \lambda_4' f' + \lambda_5 k_0 + \lambda_6 G + \lambda_6' f' Bf - \lambda_6 f'^2 (A - B)$$

where in conformity with the properties of the media being considered always $A - B > 0$, $A - 2B < 0$.

The Weierstrass-Erdmann conditions. At conjugate points of the extremal arcs the following conditions should be satisfied:

$$\Delta(F - f'F_{f'} - g'F_{g'})\delta x_c + \Delta(F_{g'})\delta f_c' + \Delta(F_{f'})\delta f_c = 0 \tag{2.17}$$

where Δ is the difference between the values in front of and behind the conjugate point with coordinates (x_c, y_c), and f_c' is the derivative at this point. If additional conditions are not imposed on the behaviour of the extremal at the conjugate point, then conditions (2.17) are rewritten thus in expanded form:

$$\Delta[\lambda_6(Af'^2+G)-\lambda_3 f'-\lambda_5 k_0]=0$$
$$\Delta[\lambda_4+Bf\lambda_6]=0,\ \Delta[\lambda_3+\lambda_5]=0 \tag{2.18}$$

Taking account of (2.6) it follows from the second relationship in (2.18) that the functions $\lambda_4(x)$ and $\lambda_6(x)$ are continuous, and since $\lambda_4(x)=Bf_c\lambda_6(x_c)$, we have

$$\lambda_4(x_c)=\lambda_6(x_c)=0 \tag{2.19}$$

On the basis of (1.11) and (2.8) it can also be concluded that

$$\lambda_4(x)=0,\ \lambda_5(x)=0,\quad \text{if } \lambda_6(x)\neq 0$$
$$\lambda_6(x)=0,\quad \text{if } \lambda_4(x)\neq 0 \ \text{ or } \lambda_5(x)\neq 0 \tag{2.20}$$

Consequently, we obtain from (2.10) and (2.11) that the extremal should be terminated by the arc

$$Af'^2+Bff''+G=0 \tag{2.21}$$

It cannot start with the arc (2.21) since $f'(0)=\infty$ for the curve (2.21), which contradicts condition (1.7). Consequently $\lambda_6(0)=0$.

Thus, we have shown that if the function $f(x)$ minimizes the functional (1.3) under conditions (1.2), (1.7)-(1.9) and given conditions at the ends of the interval, quantities, dependent on $x, w, \alpha, \beta, \lambda_3, \lambda_4, \lambda_5, \lambda_6$, satisfying Eqs. (1.11), (1.12), (2.8) and (2.13) and conditions (2.3), (2.6), (2.17) should exist.

We note that if the quantity x_k is arbitrary: we then obtain from (2.4) and (2.14)

$$c_1=0\,; \tag{2.22}$$

If the quantities S and V are arbitrary, the following respective relationships will be satisfied:

$$\lambda_1=0,\ \lambda_2=0 \tag{2.23}$$

3. Minimum resistance body shape when one of the governing geometric paraemters is specified. We consider the solution of the above variational problem for different cases of giving one of the geometric parameters governing the body shape (the volume V, the surface area S, the length L, and the maximum diameter d).

As follows from (2.5), (2.10), (2.20) and (2.19), the conditions

$$\lambda_6(x_k)=-f_k(f_k'+\mu_0)$$
$$\lambda_4(0)=\lambda_6(0)=0,\ \lambda_4(x_c)=\lambda_6(x_c)=0 \tag{3.1}$$

should be satisfied in all cases listed.

For cases when the body diameter d is arbitrary, we obtain from (2.3) and (2.4)

$$[Ff_k']=[\lambda_3+\lambda_5+\lambda_7]_k=0 \tag{3.2}$$

A solution of the problem with $f(x)\not\equiv 0$ does not exist in those cases when one geometric body parameter is given.

Indeed, we assume condition (2.10) satisfied; we then obtain (2.12). The extremal $f(x)$ is terminated by a segment of the arc (2.21). But it then follows from (3.2) and (2.20) that

$$\lambda_3(x_k)=0,\ \ \lambda_5(x_k)=0 \tag{3.3}$$

Substituting (3.3) and the first condition from (3.1) into (2.15), independently of the assignment of the parameter L, V or S, we obtain that

$$c_1=\lambda_1=\lambda_2=0 \tag{3.4}$$

We obtain from (2.15) for the arc (2.21)

$$\lambda_3 g=-Bfg'[(g+\mu_0)f+\lambda_6] \tag{3.5}$$

Since $\lambda_5(x)\equiv 0$ on the arc (2.21), then from the second equation of (2.13) and (3.5) we obtain for the function $\lambda_3(x)$ on the arc (2.21)

$$\lambda_3 f=A_0=\text{const} \tag{3.6}$$

Under the conditions (3.4), Eq. (3.6) is always satisfied on the arc (2.21). Taking (3.3) into account we obtain from (3.6)

$$\lambda_3(x)\equiv 0 \tag{3.7}$$

Since $\mu_0>0, f'\geqslant 0$, then it follows from (3.5) and (3.7) that the third condition in (3.1) can only be satisfied if $f_c=0$. And, in turn, this is only possible when the extremal is the arc

$$f\equiv 0 \tag{3.8}$$

The result obtained contradicts condition (2.10).

Therefore, the extremal should satisfy condition (2.9) when one of the parameters L, S or V is given. In these cases the arc (3.8) will be a solution of the problem if the boundary conditions on L, S or V are satisfied here. Otherwise, no solution of the problem exists.

When just the body length L is given, an arc (3.8) of length L will be the solution. The single boundary condition is satisfied here. When $S\neq 0$ or $V\neq 0$ is given, the arc (3.8) cannot be a solution of the problem since the conditions on S and V are not satisfied. Therefore, no solution of the problem exists in these cases.

We now examine the case of giving just the maximum body diameter $d = 2R_0$. Since L, S and V are arbitrary here from (2.22)-(2.23), conditions (3.4) will be satisfied. Since $d \neq 0$, condition (2.10) is satisfied; therefore, a segment of the arc (2.21) will be the last section of the extremal. On this arc the function $\lambda_3(x)$ satisfies Eq.(3.6). Taking account of (3.1) we obtain from (3.5)

$$[\lambda_3 f']_k = 0 \tag{3.9}$$

As was noted above, if $\lambda_3(x_k) = 0$, then condition (2.10) will be violated, and we therefore obtain from (3.9)

$$f_k' = 0 \tag{3.10}$$

Let us find the shape of the extremal for a given diameter d. The extremal satisfies (2.16), which taking (3.4) into account we can write in the form

$$\begin{gathered}[2(A-B)f'^3 + \mu_0(A-2B)f'^2 - \mu_0 G]f = \\ \lambda_4' f' - \lambda_5 k_0 + \lambda_6 G + \lambda_6' ff'B - \lambda_6 f'^2 (A-B)\end{gathered} \tag{3.11}$$

It hence follows that the extremal cannot contain the arc along which $w \neq 0$ and $\beta \neq 0$. Otherwise we will have from (2.8) $\lambda_4 = \lambda_5 = \lambda_6 = 0$. But under this condition we obtain that a cone with $w = 0$ will be the solution (3.7), which contradicts the assumption made. Therefore, the desired extremal can consist of just two arcs $f'' = 0$ and $Af'^2 + Bff'' + G = 0$, where, as noted at the end of Sect.2, it can only start with the former of these arcs and terminate with the latter.

Now let k_0 be a known constant. We will show that depending on the satisfaction of the conditions

$$\Phi(k_0) = 2(A-B)k_0^3 + \mu_0(A-2B)k_0^2 - \mu_0 G \geqslant 0 \tag{3.12}$$

$$\Phi(k_0) < 0 \tag{3.13}$$

two classes of solutions are obtained.

If condition (3.12) is satisfied, then $\lambda_5(x) \equiv 0$ along the extremal.

Indeed, if $\lambda_5 \not\equiv 0$, the extremal starts with a generator of the cone $f = k_0 x$, where the left side of Eq.(3.11) is greater than zero. But then if the limits as $f \to f_c^+$ are considered in (3.11), then taking (3.1) and (2.8) into account we obtain that $\lambda_6'(x_c) > 0$. And since $\lambda_6(x_c) = 0$, the condition obtained will contradict (2.6). Therefore, in this case the extremal cannot start with the cone generator $f = k_0 x$ and consequently $\lambda_5(x) \equiv 0$.

Then as follows from (3.11), we have on the arc $f(x) = kx$

$$\lambda_4'(x) = [2(A-B)k^3 + \mu_0(A-2B)k^2 - \mu_0 G]x \tag{3.14}$$

i.e., $\lambda'(x)$ is a linear function where $\lambda_4'(0) = 0$. If $\lambda_4'(x) \not\equiv 0$, we obtain that it is impossible to satisfy the conditions $\lambda_4(0) = 0$, $\lambda_4(x_c) = 0$, where $x_c \neq 0$.

Therefore, it is shown that under the conditions (3.12) the extremal starts with the cone generator $f = kx$, where k is determined from the condition $\lambda_4(x) = 0$ or $\Phi(k) = 0$, or

$$2(A-B)k^3 + \mu_0(A-2B)k^2 - \mu_0 G = 0 \tag{3.15}$$

Now, let condition (3.13) be satisfied. Then since the condition $k \leqslant k_0$ is satisfied for allowable k, condition (3.13) will be satisfied for all possible k according to the general form of the function $\Phi(k)$. If it is assumed that $\lambda_5 \equiv 0$ along the extremal, then the left side of (3.11) is less than zero for all $f > 0$. This means that we have $\lambda_4'(x) < 0$ on the arc $f = kx$ for $x > 0$, which contradicts the conditions $\lambda_4(0) = 0$ and $\lambda_4(x) \geqslant 0$. Therefore, the assumption that $\lambda_5 \equiv 0$ is not true. Consequently, in the case when condition (3.13) is satisfied, the extremal should start with the arc

$$f = k_0 x \tag{3.16}$$

When either of conditions (3.12) and (3.13) is satisfied, the extremal should be terminated by a segment of the arc (2.21).

Let us find the ordinate of the conjugate point of this arc and the cone generator $f = kx$. To do this we solve the differential Eq.(2.21) by using condition (3.10). Then

$$f' = \left[\left(\frac{C_0}{f}\right)^\alpha - \frac{G}{A}\right]^{1/2}, \quad C_0 = R_0\left(\frac{G}{A}\right)^{1/\alpha}\left(\alpha = \frac{2A}{B}\right) \tag{3.17}$$

At the conjugate point $f_c' = k$, consequently, we obtain from (3.17)

$$f_c = C_0\left(k^2 + \frac{G}{A}\right)^{-1/\alpha} = R_0\left(\frac{A}{G}k^2 + 1\right)^{-1/\alpha} \tag{3.18}$$

We obtain an explicit form of the arc (2.21) from (3.17) in the form

$$x = \int_{x_c}^{R_0} \left[\left(\frac{C_0}{f} \right)^{\alpha} - \frac{G}{A} \right]^{-1/2} df + c_1 \tag{3.19}$$

where the constant c_1 is determined from the coordinates, known from (3.19) for the conjugate point $(x_c = f_c/k,\ f_c)$. Hence, (3.15) or (3.16) and (3.19) define the extremal completely when the maximum body diameter d is given. The general form of the extremal is presented in Fig.1.

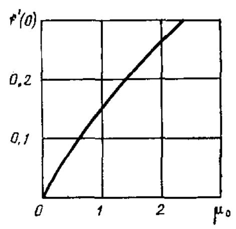

Fig.2

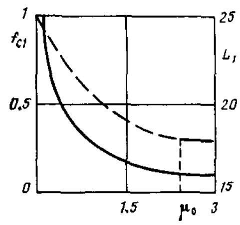

Fig.3

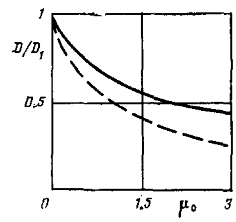

Fig.4

4. Examples of the computation.

The extremal body shape was computed on a BESM-6. A model medium was taken for which $\theta = 20°$, $m = 0.5$ kg/cm^2, $\rho_0 = 153$ kg/m^3, $p_a = 3$ kg/cm^2, $b_1 = 0.6$ for a $u = 600$ m/sec body velocity. The critical value is $k_0 = 0.3$, $d = 2$.

Fig.2 shows results of computing $f'(0)$ as the coefficient of friction μ_0 changes. The critical value μ_0^* for which $f'(0) = k_0$, equals 2.4. For $\mu_0 > \mu_0^*$ the apex angle of the conical leading part of the extremal body does not change. As computations showed (Fig.3), the length of a minimum-resistance body $L_1 = L/R_0$ (the solid line) and the conjugate ordinate $f_{c_1} = f_c/R_0$ (the dashed line) diminish as the coefficient of friction μ_0 increases.

Comparison of the magnitude of the resistance of the extremal body obtained with the resistance of a cone that is optimal among all cones for a given dimensionless body diameter $d = 2$ of interest.

The shape of this optimal cone can be found from (1.3) by substituting $f = kx$ and then determining the minimum resistance as k varies. For the quantity k for an optimal cone we obtain the expression

$$k^{-1} = (q + r)^{1/3} + (q - r)^{1/3} \tag{4.1}$$

$$q = A/(\mu_0 G),\quad p = A/(3G),\quad r = (q^2 + p^3)^{1/2}$$

The resistance of the cone is defined by the expression

$$D_1 = \pi R_0^2 (Ak^2 + \mu_0 Ak + G + \mu_0 G/k) \tag{4.2}$$

The ratio between the resistance D of the extremal body and the resistance D_1 of the optimal cone is presented in Fig.4 (the solid line) as a function of the quantity μ_0; the ratio between the magnitude of the resistance of the extremal body and the resistance of a cone when the bodies being compared have identical length and base diameter is shown by dashes. It is seen that the optimal bodies possess considerably less resistance than the conical bodies.

We note that the programs compiled for the BESM-6 when carrying out the research enable one to solve the optimal optimization problem for any given value of the body parameters and the plastically compressible medium.

5. On minimum resistance bodies moving at constant velocity in an elastic-plastic medium.

The methods of analysis developed in Sects.1-4, obviously carry over entirely to the case of the motion of thin bodies of revolution at constant velocity in other media when the hypothesis of plane sections can be considered valid /1/.

In particular, the analysis performed and the computation programs developed for the BESM-6 hold for elastic-plastic media simulating metals, for which the resistance D is also determined by the functional (1.3) with the coefficients /1/.

$$A = \chi \left[\ln(1+\varepsilon) - \frac{\varepsilon}{1+\varepsilon} \right],\quad B = \chi \ln(1+\varepsilon) \tag{5.1}$$

$$G = \tau\,[1 + \ln(1+\varepsilon)];\quad \varepsilon = E/[2\tau(1+\nu)]$$

where τ is the yield point of the medium during shear, E is Young's modulus, and ν is Poisson's ratio.

REFERENCES

1. GENDUGOV V.M., ROMANOVA S.V. and ROMODANOVA T.V., Minimum resistance solid of revolution in elastic-plastic and plastically compressible media. Problems of the Dynamics of Interaction of Deformable Media, Izd. Akad. Nauk SrmSSR, Erevan, 1984.
2. RAKHMATULIN KH.A., SAGOMONYAN A.YA. and ALEKSEYEV N.A., Problems of Soil Dynamics, Izd. Mosk. Gos. Univ., 1964.
3. SAGOMONYAN A.YA., Pentration. Izd. Mosc. Gos. Univ., Moscow, 1974.
4. IOFFE A.D. and TIKHOMIROV V.M., Thoery of Extremal Problems. Nauka, Moscow, 1974.

Translated by M.D.F.

PMM U.S.S.R.,Vol.51,No.3,pp.392-399,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

ON THE CORRESPONDENCE PRINCIPLE IN THE PLANE CREEP PROBLEM OF AGEING HOMOGENEOUS MEDIA WITH DEVELOPING SLITS AND CAVITIES*

S.A. NAZAROV, L.P. TRAPEZNIKOV and B.A. SHOIKHET

The plane creep problem of ageing homogeneous media is considered. The bulk and shear relaxation kernels are assumed to be distinct. Bulk forces, temperature deformations and stresses prescribed on the whole body boundary are the actions. Representations are obtained for the stress, strain, and displacement in terms of the solution for elasticity theory problems for a domain with a fixed boundary and with slits and cavities growing according to a given law.

For a domain with a moving crack it is proved under certain constraints /1/ that the stresses in the creep problem agree with the stresses in the elasticity problem. For a domain with a fixed boundary, necessary and sufficient conditions are obtained /2/ for agreement between the stresses of the creep and elasticity problems. For a constant Poisson's ratio the problem being studied /3/ is investigated in a more general formulation.

A survey of the research devoted to the correspondence principle in the creep theory of ageing media is presented in /4/.

1. Let a homogeneous isotropic linearly-deformable body possessing the properties of ageing and creep occupy a two-dimensional domain $\Omega(\tau) = \Omega_0 \setminus (\bar{\omega}(\tau) \cup \gamma(\tau))$ ($\bar{\omega}$ is the closure of the domain ω). Here $\tau \in [0, t]$ is the time, Ω_0 is a fixed bounded simply-connected domain, and $\omega_i(\tau)$, $\gamma_j(\tau)$ are quasistatic growing (i.e. $\Omega(\tau_1) \subset \Omega(\tau_2)$ for $\tau_1 > \tau_2$) cavities and slits with given laws of growth

$$\omega(\tau) = \bigcup_{i=1}^{N} \omega_i(\tau), \quad \gamma(\tau) = \bigcup_{i=N+1}^{N+J} \gamma_i(\tau)$$

It is assumed that $\omega_i(\tau)$ are simply-connected domains with piecewise-smooth boundaries $\partial\omega_i(\tau)$ /5/, while $\gamma_i(\tau)$ are simple unclosed curves made up of the smooth arcs $\bar{\omega}_i \cap \bar{\omega}_j = \Lambda$, $i \neq j$, $i, j = 1, \ldots, N$, $\gamma_i \cap \gamma_j = \Lambda$, $i \neq j$, $i, j = N+1, \ldots, N+J$ and given parametrization $\mathbf{x}_j(\zeta, \tau)$, for $\zeta \in [0, 1]$, of the curves $\partial\omega_i(\tau), \gamma_j(\tau)$ and piecewise-continuous in τ.

The boundary $\partial\Omega(\tau)$ of the domain $\Omega(\tau)$ consists of the boundary $\partial\Omega_0$, the cavity boundaries $\partial\omega_i(\tau)$ and the edges $\gamma_j^{\pm}(\tau)$ of the slits $\gamma_j(\tau)$. The bulk forces $\mathbf{f} = \{f_i(\mathbf{x}, \tau)\}$ and the temperature $T(\mathbf{x}, \tau)$ are given for $\mathbf{x} \in \Omega(\tau)$, $i = 1, 2$; the surface loads $\mathbf{F} = \{F_i(\mathbf{x}, \tau)\}$ are defined for $\mathbf{x} \in \partial\Omega(\tau)$, and equilibrium conditions are satisfied for all τ.

The equations of the plane creep problem have the form

$$\varepsilon_{ij}(\mathbf{x}, \tau) = 2^{-1}(u_{i,j}(\mathbf{x}, \tau) + u_{j,i}(\mathbf{x}, \tau)), \quad i, j = 1, 2, \quad \mathbf{x} \in \Omega(\tau) \tag{1.1}$$

**Prikl.Matem.Mekhan.*,51,3,504-512,1987